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FRACTURE BY FATIGUE

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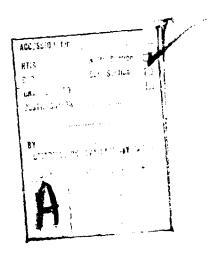
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The lifetime of manufactured items under intended service conditions is often limited by the processes of corrosion, wear and fatigue. Since such processes represent economic loss and perhaps a safety hazard to the consumer, much effort has gone into the understanding of these phenomena as well as into improved design procedures to guard against their occurrence. In the field of fatigue in particular a considerable advance in recent years has been made in the quantitative treatment of the fatigue process, especially with respect to (cont'd on reverse side)

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20. the matter of fatigue crack growth. Improved understanding of the fatigue crack process is timely as in certain circumstances, as for example in the case of welded structures, it is not the initiation of cracks but rather the growth of cracks from pre-existing defects which is the critical aspect in determining service lifetime. Other advances have been made in improving the resistence of materials to fatigue either through the control of chemistry or by control of processing variables. Such procedures are generally more important in affecting the crack initiation rather than the crack propagation stages. In this presentation a review of the current status of fatigue will be given from the mechanistic as well as the design viewpoints. Areas in need of further understanding such as corrosion fatigue, creep-fatigue, and fatigue under variable amplitude loading will also be considered.

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## FRACTURE BY FATIGUE

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presented at the

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DEFINITION OF THE PROBLEM

at the

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## Introduction

In this paper we are concerned with the problem of fatigue and the steps being taken to minimize its occurrence. Fatigue failure, of course, is due to the repeated application of stress. This type of failure can occur in crystalline as well as non-crystalline materials, with metals and polymers being of principal concern. Fatigue failure involves both the initiation and propagation of cracks, and irreversible plastic deformation plays a key role in both, and it may be surprising, therefore, that a completely elastic, i.e., brittle material, is not subject to fatigue. However, such materials are not generally useful as engineering materials, a notable exception being a composite material which contains brittle fibers encased in a protective matrix. Therefore, although ductility is a highly desired characteristic of a metal, under cyclic loading it leads to failure. Perhaps Aesop could find a moral here.

In considering the various components subject to fatigue, sometimes a division of these commonents into two categories is made. The first of these are components which comprise the primary load bearing members of structures such as airplanes, cars, trucks, bridges, pressure vessels, etc. The second category includes the remainder of products subject to fatigue, a category which includes the bulk of mechanical products ranging from can openers to aircraft turbine blades. This latter category can also be broadened to include certain types of wear failures which in fact result from contact fatigue. Of these two categories, more attention has been given in general to structural failures due to fatigue. In addition to fatigue, other forms of structural failure are: overload failure of a ductile nature, overload failures of a brittle nature, creep, stress corrosion and

instability. Of these failure types fatigue is the most common form of failure simply because few structures are subjected to the static loading assumed in design. (1) In fact Freudenthal (2) states that 95% of all structural failures are fatigue failures. Structural fatigue failures often attract considerable attention because of their sometimes catastrophic nature, which result in loss of life as well as economic losses, as in the case of the Comat failures. Non-structural failures are usually less dramatic, but nonetheless the integrated economic loss is undoubtedly significant as is the drain on natural resources as a result of such failures, although statistics on the frequency of occurrence as well as the economic factors associated with such losses are not generally available. In any event, such figures must be carefully interpreted, for a low frequency of occurrence may not necessarily indicate an absence of a fatigue problem but may instead reflect a competent handling of the problem. Such is the situation in the transport aviation field in recent years, where improvements in fatigue analysis, fatigue testing, and in the capabilities of non-destructive inspection have served to control the fatigue problem. In other areas of the consumer product field, design against fatigue may not be as carefully carried out and fatigue can be a more common occurrence. Some 700,000 product liability law suits are currently pending against manufacturers, (3) an increase by an order of magnitude in a span of a few years. (4) Such an increase reflects a growing expectation for product reliability and safety on the part of the consumer. Undoubtedly many of these failures, rightly or wrongly, are attributed to fatigue resulting from poor design.

Fatigue has now been the subject of research investigations for over one hundred years, and despite the progress made, failures continue to occur. This situation is due in part to the complex nature of the fatigue process and the stress-material-environmental interactions involved therein. However, much of the fatigue problem is simply due to poor communications. Many designers do not know how to deal properly with fatigue and do not treat it as a matter of concern until product failures start to occur. This situation can lead to a rapid but expensive development of an appreciation of the complex nature of fatigue and of the procedures available to aid in avoiding such failures. This is not to say that we can now design Holmes' one-horse shay, or match the capabilities of Neville Shute's hero who accurately predicted that the plane on which he was a passenger was about to experience fatigue failure, but we are getting closer. Today there are two approaches to design against fatigue. (5) One of these is to design for a safe life in terms of the S-N properties of the material. The other approach is to design damage tolerant (fail-safe) structures so that in the event that fatigue cracks do develop their presence will not be catastropic. Non-destructive inspection is an important aspect of this latter approach. The purpose of the remainder of this paper will be to review the basic aspects of these two design approaches, with emphasis placed upon the unit processes of crack initiation and growth in simple specimens. Knowledge of these processes is important in the design stage, with simulated service testing being relied upon to provide a final check of the fatigue analysis of many complex structures.

### Lifetime Determination

A representative S-N curve is shown in Fig. 1. (6) To emphasize that scatter of test results is an important characteristic of fatigue, survival probability estimates are also included. The safe-life design approach uses such curves, coupled with component testing to establish fatigue reliability, particularly in the high cycle range.

Fatigue is generally a surface sensitive phenomenon with cracks in polished specimens initiating at sites such as slip bands in or at inclusions, Fig. 2, depending upon the nature of the metal. In total life determination of simple specimens, the distinction between crack initiation and propagation in the failure process is not made, at least at the design level. Cracks of the order of a millimeter in size are usually visible only in the last stages of life, although they may be present but of smaller size for a much greater portion of the total lifetime, especially for lifetimes less than 1000 cycles. This total lifetime will be influenced by the environment. For example, at elevated temperatures oxidation can exert a deleterious effect, as shown in Fig. 3. (7) Note that in this case the fatigue lifetime is plotted as a function of plastic strainrange, rather than stress amplitude. At elevated temperatures the time dependent nature of deformation involving creep or stress relaxation can also become important. Fig. 4 shows a variety of hysteresis loop shapes as well as a thermal fatigue cycle. (8) Analysis of cyclic loading under such conditions is obviously complex, but new methods of analysis to deal with such loading histories are being developed, as for example, the strainrange partitioning methods due to Manson and co-workers. (9) The importance

of surface finish on fatigue is indicated in Fig. 5. (10) Allowance for scatter, surface finish, corrosion, and in-service surface damage can be combined in certain instances to reduce the allowable design stress by as much as a factor of ten below the average smooth specimen stress for a given lifetime.

In the low cycle range where inelastic deformation is dominant it is more usual to relate the lifetime to a strain range rather than a stress range because relatively simple relationships, as indicated in Fig. 3, can exist between plastic strainrange and lifetime stress amplitude for fully reversed strain. These relationships are as follows, using the notation of Morrow: (11)

$$\frac{\Delta \varepsilon_{\mathbf{p}}}{2} = \varepsilon_{\mathbf{f}}^{\dagger} (2N_{\mathbf{f}})^{\mathbf{c}} \tag{1}$$

$$\frac{\Delta \varepsilon_{\mathbf{p}}}{2} = \frac{\sigma_{\mathbf{f}}^{\mathbf{e}}}{E} (2N_{\mathbf{f}})^{\mathbf{b}}$$
 (2)

$$\sigma a = K' \left(\frac{\Delta \varepsilon_{\mathbf{p}}}{2}\right)^{n'} \tag{3}$$

where  $~\Delta\epsilon_{D}^{}$  is the plastic strainrange

 $\epsilon_{\mathbf{f}}^{\dagger}$  is the fatigue ductility coefficient

 $2N_{\mbox{\scriptsize f}}$  is the number of reversals to failure

c is the fatigue ductility exponent

 $\Delta\epsilon_{\mathbf{p}}$  is the elastic strainrange

 $\sigma_{\mathbf{f}}^{\bullet}$  is the fatigue strength coefficient

b is the fatigue strength exponent

 $\sigma_n$  is the true stress amplitude

K' is the cyclic strength coefficient

n' is the cyclic strain hardening exponent

E is the elastic modulus

Equations (1) and (2) can be combined to yield

$$\frac{\Delta \varepsilon_{\mathbf{t}}}{2} = \frac{\Delta \varepsilon_{\mathbf{e}}}{2} + \frac{\Delta \varepsilon_{\mathbf{p}}}{2} = \frac{\sigma_{\mathbf{f}}}{E} (2N_{\mathbf{f}})^{\mathbf{b}} + \varepsilon_{\mathbf{f}}^{\dagger} (2N_{\mathbf{f}})^{\mathbf{c}}$$
(4)

Maraging steel is an example of an alloy which follows these average lift ime relationships is shown in Fig. 6. (12) High strength aluminum alloys and titanium alloys generally do not exhibit such good agreement. In such cases a more complete testing program must be employed to obtain reliable fatigue life relationships.

Design situations often involve a mean stress whereas much fatigue test data, particularly those obtained in the low-cycle range, are obtained under fully reversed cycling. A Goodman diagram, Fig.  $7^{(13)}$  for example, can be used to determine the allowable stress amplitude for a given lifetime,  $\sigma_a$ , at any mean stress,  $\sigma_m$ . If it is assumed that the stress amplitude for a given lifetime varies linearly with mean stress then we can write

$$\sigma_{\mathbf{a}} = \sigma_{\mathbf{a}} \left( 1 - \frac{\sigma_{\mathbf{m}}}{\sigma_{\mathbf{u}}} \right) \tag{5}$$

where  $\sigma_{\overline{a}}$  is the stress corresponding to a given lifetime for fully reversed loading, and  $\sigma_u$  is the ultimate tensile strength. For some materials it may be preferable to use the true stress at failure,  $\sigma_f$ , in place of  $\sigma_u$ .

Another design complication is that the in-service loading is rarely of the constant amplitude variety. Random loading has to be considered, and a first problem is the selection of a statistical counting method assuming that siequate load data is available. Current statistical counting methods are modifications of the three basic types indicated in Fig. 8. (14) These three types are characterized by:

- a. The variable stress reaches a maximum or a minimum.
- b. The variable stress changes from a minimum to a maximum, i.e., it describes a positive or negative range.
- c. The variable stress crosses a given level in a positive or negative direction.

Recent methods such as the rain-flow and range-pair cycle counting methods are variations of type b in which a second counting condition, the instantaneous mean value is introduced. Fig. 9 is a strain time history in which a pairing of ranges to form a cycle is made. For example, in Fig. 9, a range is counted between peak 1 and peak 8, and each range that is counted is paired with the next straining of equal magnitude in the opposite direction to make up a complete cycle. In Fig. 9 part of the range between peaks 8 and 9 is paired with the range counted between peaks 1 and 8. In Fig. 9 the counted ranges are marked with solid lines and the paired ranges with dashed lines. Each peak is taken in order as the initial peak of a range, except that a peak is skipped if the part of the history immediately following it has already been paired with a previously counted range. (15)

Each strainrange can be converted to an equivalent strainrange for fully reversed loading by using Eq. 5. The fraction of life expended,  $\frac{1}{N_1}$ , for each cycle can then be computed. Miner's law,  $\sum_{i=1}^{i=1} \frac{n_i}{N_1} = 1$ , is then used to predict the total number of cycles which can be applied before failure. Dowling (15) used this approach to predict the fatigue life of 2024 - T4 aluminum alloy under a complicated stress-strain history and found predicted lives to be within a factor of three of the actual lives in 83 tests.

Thus far, we have considered lifetime prediction for simple specimens and modifications due to effects of surface funish, mean stress and candom loading. Another important consideration is the effect of a not h on fatigue properties. In cases where Eqs. 1, 2, and 3 hold they can be used in conjunction with Neuber's rule  $^{(16)}$  (to be defined) to . predict the fatigue lifetime as a function of the applied R ratio (R being the ratio of minimum to maximum stress in a simple loading cycle). In considering the plastic behavior at the root of a notch it can be shown that the local R ratio differs from the macroscopic value of  $^{(17)}(18)$ . For example, under R = 0 loading, the value of R at the notch root may be equal to -1. In order to estimate the local R value use is made of the Neuber rule:  $^{(16)}$ 

$$\kappa_{\rm T}^2 = \kappa_{\rm \sigma} \kappa_{\rm \epsilon}$$
 (6)

which leads to

$$K_{\rm T}^2 \frac{{\rm S}^2}{{\rm E}} = \sigma_{\ell} \epsilon_{\ell} \tag{7}$$

where  $K_{\text{T}}$  is the theoretical stress concentration factor

 $K_{\sigma}$  = the local stress concentration factor

K = the local strain concentration factor

S - the applied elastic stress

 $\sigma_{\ell}$  = the local peak stress at the notch

 $\epsilon_{\varrho}$  = the local peak strain at the notch

To take into account notch sensitivity and notch size effect (an effect related to the volume of material subject to the peak stress at the tip of a notch), Eq. 7 can be written as

$$K_{\overline{F}}^{2} \frac{S^{2}}{E} = \sigma_{\ell} \varepsilon_{\ell} \tag{8}$$

where  $K_{\overline{F}}$  is the fatigue stress concentration factor determined at lifetimes such as  $10^7$  and defined as the ratio of the unnotched to notched fatigue strengths for fully reversed loading.

Fig.  $10^{(19)}$  is a simplified version of the stress-strain behavior at the root of a notch when yielding occurs on the tensile part of a cycle and is based upon the experimental work of Wetzel<sup>(17)</sup> and Crews and Hardrath<sup>(18)</sup>.

Again, Eq. 5 can be used to convert a stress amplitude at a given stress to an equivalent stress or strain amplitude fully reversed loading or vice versa. To obtain the R=-1 baseline information, two approaches can be used. One is to use Eqs. 1, 2 and 3 together with the Neuber rule, Eq. 6, to obtain

$$S_{\overline{a}} = \frac{\sqrt{K^*E}}{K_{\overline{f}}} \left\{ \left[ \varepsilon_{f}^* (2N_f)^c \right]^{n^*+1} + \frac{\sigma_{f}^*}{E} (\varepsilon_{f}^*)^{n^*} (2N_f)^{2b} \right\}^{1/2}$$
 (9)

where  $S_{\overline{a}}$  is the applied stress amplitude for fully reversed loading.

Alternatively, one can use the fatigue data for unnotched specimens tested under fully reversed loading in the nominally elastic range. To obtain a value of  $S_{\overline{a}}$  for any lifetime, simply divide the unnotched stress value by  $K_{\overline{N}}$ . Then to obtain the value of  $S_{\overline{a}}$  for that lifetime corresponding to the local R value, reduce  $S_{\overline{a}}$  by the ratio of  $\sigma_{\overline{a}}/\sigma_{\overline{a}}$  given in Eq. 5.

This latter procedure was followed to prepare the curves shown in Fig. 11.

For the low stress range the local yield stress is not exceeded and the following relation is used:

$$K_{\overline{F}_{0}} = K_{\overline{F}} \left( \frac{\sigma_{u} + \frac{\sigma_{\overline{a}u}}{K_{\overline{F}}}}{\sigma_{u} + \sigma_{\overline{a}u}} \right)$$
 (10)

where  $K_{F_0}$  is the fatigue notch factor for R = 0 loading, and  $\sigma_{\overline{a}u}$  is the fatigue strength of an unnotched specimen at a specified number of cycles for R=-1 loading. It is noted that as the applied stress increases the local R value changes rapidly and takes on a value of -1.

These are the basic ingredients of the safe-life design approach. In certain cases other factors may also have to be considered. These may include fretting fatigue, contact fatigue, corrosion fatigue, and cyclic stability, for example. The principal subdivision of the safe-life approach is based upon consideration of whether the fatigue process is of a low cycle or high cycle nature. A distinction between these two can be made by comparing the elastic and plastic strain amplitudes. The low cycle fatigue region is that region in which the plastic strain amplitude is larger than the elastic, and the high cycle fatigue region exists where the elastic strain amplitude is larger than the plastic. At some strain the elastic and plastic strain amplitudes are equal as seen in Fig. 6. Fig. 12<sup>(20)</sup> summarizes the important characteristics associated with low and high cycle fatigue as related to the transition lifetime.

#### FATIGUE CRACK PROPAGATION

Consideration of fatigue crack propagation is important in the damage-tolerant approach to fatigue design. An analytical approach to crack propagation can be obtained by the application of fracture mechanics concepts to this subject. The most important aspect of the use of fracture mechanics is the single-valued correlation in the linear elastic range between the stress intensity factor and the rate of fatigue crack growth,  $\Delta a/\Delta N$ , where a is the crack length and N is the number of cycles. The stress intensity factor, K, is related to the stress concentration factor, K, through the following definition of the stress intensity factor.

$$K = \lim_{\rho > 0} K_{\sigma} \sqrt{\frac{\pi \rho}{2}}$$
 (11)

with  $K_{\sigma}$  and  $\sigma$  based upon the gross cross-sectional area, and  $\rho$  is the tip radius. For the case of a central slit in a sheet specimen subjected to tensile loading at right angles to the slit the expression for K (with the crack length much smaller than the specimen width) becomes

$$K = \sigma \sqrt{\pi a} \tag{12}$$

In general the stress intensity will be of this form, but modified to account for particular geometry, in which case we can write

$$K = \sigma \sqrt{\alpha \pi a}$$
 (13)

where  $\alpha$  is a factor which takes into account the particular geometry. Consideration is now being given to the effect of structural modifications such as stringers on the rate of crack growth, however the bulk of research in the past has involved specimens of constant thickness in which material response rather than structural response can be established.

The characteristic dependence of the rate of fatigue crack growth on the stress intensity factor is indicated in Fig. 13. There are two abyourtotic limits to the curve. The upper limit is set by the fracture toughness of the material, K. The lower limit is referred to as the threshold for crack growth,  $\mathbf{K}_{\mathrm{TH}}$ . This latter quantity is currently of considerable interest, for a new design philosophy based upon the quantity is emerging. Many parts before going into service already contain crack-like defects as in the case of welded joints. Some of these parts may have to be designed for infinite safe-life. In the absence of defects this would entail designing at stresses based on the 10 life of the S-N curve, but in the presence of defects the new approach is to insure that the stress intensity associated with defects is kept below the threshold level for crack growth. Wells (21) has considered the implications of this approach for both internal and surface flaws. The present capability for the detection of internal flaws varies from 0.5 mm up to 5 mm depending upon particular conditions. If design were based upon the maximum size flaw in the most critical position the allowable stresses would be unacceptably low. Rather than use such low stresses, material processing is controlled so that the probability of such damaging defects is extremely low. The capability for detection of surface cracks is much greater than for internal cracks, with surface crack lengths of 0.05 mm being within the range of current capabilitites. For these smaller flaws the fracture mechanics approach could be used in the control of crack growth even above the threshold level. However, for these small surface cracks there may still be

difficulties with the use of the fracture mechanics approach as indicated in Fig. 14<sup>(22)</sup> where it is seen that surface flaws grow more rapidly than do through-cracks at low stress-intensity values. Such a lack of correlation with the stress intensity factor suggests that the continuum approach may break down for crack sizes comparable to metallurgical features such as grain size, for example. Application of the fracture mechanics approach above the threshold level will be therefore more straightforward in dealing with structures which contain cracks of more significant size. For example, the large scale of an aircraft wing structure favors the approach, where the small scale of an aircraft turbine blade may restrict the approach.

The rate of fatigue crack propagation can be expressed as a function of the stress intensity factor, but to put such an expression on a rational basis, consideration should be given to the modes of separation involved. At low crack growth rates, i.e., below 10<sup>-4</sup> in at R=0, a ductile mode of crack advance associated with plastic blunting on loading, and tip sharpening on unloading as illustrated in Fig. 15 is thought to be dominant. (23) At higher rates of crack growth where the peak stress intensity approaches the fracture toughness, static modes of separation such as ductile rupture become operative and accelerate the rate of crack growth. The following expression based on crack-opening displacement considerations has been developed (19) to account for the contribution of each of these modes to the amount of crack growth per cycle:

$$\frac{\Delta a}{\Delta N} = \frac{A}{\sigma_y E} \left( \Delta K^2 - \Delta K^2_{TH} \right) \left( 1 + \frac{\Delta K}{K_c - K_{max}} \right)$$
 (14)

Since  $K_{max} = \frac{\Delta K}{1-R}$ , the effects of mean stress are incorporated in the equation. Figs. 16 and 17 show a comparison with predicted and experimental results for two aluminum alloys of different toughness levels. Note that mean stress effects are much more pronounced in the case of the low toughness alloy, and they are virtually absent in the high toughness alloy. The threshold level in both alloys is sensitive to mean stress, and current research is aimed at understanding this dependency. Crack growth rates near the threshold are of the order of  $10^{-8}$  inches per cycle or less, representing increments of growth on the order of the interatomic spacing. Further, since oxides can form very rapidly in metals tested in air, the growth must be occurring primarily in the surface oxide at these low rates rather than in the metal itself. The appearance of the fracture surface should, therefore, be influenced since oxide rupture is a brittle process compared to the base metal blunting process. The characteristic fatigue striations are not observed at such low growth rates since they are resolvable only at crack growth rates above 10<sup>-6</sup> inches per cycle, but even in this range their ease of detection will be material dependent.

The effects of variable amplitude loading on the rate of fatigue crack growth are of interest. Barsom (24) has recently reported on crack growth tests of a A514-B steel (yield strength 129 KSI) subjected to random loading. He found that the rate of crack growth could be expressed as

$$\frac{\Delta a}{\Delta N} = A' (\Delta K_{rms})^{n}$$
 (15)

where A' and n are material constants, and  $\Delta K_{rms}$  is the root-mean-square stress-intensity-factor fluctuation. It is of interest to compare predictions based upon Eq. 14 with this result. Since Barsom observed that crack growth in this steel was independent of mean stress for crack growth less than  $10^{-4}$  inches per cycle, the contribution due to static modes in Eq. 14, i.e., the  $\frac{\Delta K}{K_{c}-K_{max}}$  term, can be neglected. An average

rate of crack growth under variable amplitude loading,  $(\frac{\Delta a}{\Delta N})_{AVG}$ , can be taken to be equal to the average contribution from each of the cycles over the increment of crack growth considered, that is

$$\left(\frac{\Delta a}{\Delta N}\right)_{AVG} = \frac{A}{\sigma_{V}E} \sum_{i=N}^{i=N+\Delta N} \frac{\Delta K_{i}^{2}}{\Delta N} - \Delta K_{TH}^{2}$$
 (16)

Note that the quantity  $\sum \Delta K^2$  is the square of  $\Delta K_{rms}$ , so that Eq. 16 can be written

$$\left(\frac{\Delta a}{\Delta N}\right)_{AVG} = \frac{A}{\sigma_{y}E} \left(\Delta K_{rms}^{2} - \Delta K_{TH}^{2}\right) \tag{17}$$

Fig. 18 shows a comparison between Eq. 15 and Eq. 17 and the data obtained by Barsom. (In evaluating Eq. 17, A was taken to be 0.023 and ΔK<sub>TH</sub> to be 8 ksi√in). Agreement of both equations is quite good, however, for other more extreme types of loading the agreement may not be as good. For example, a single overload can greatly retard the rate of crack growth rate at a lower amplitude, and the above equations would over-estimate the rate of crack growth. Nevertheless, for a large number of random cycles, the amplitudes of which do not fluctuate too widely, the agreement shown above indicates that reasonable predictions can be made.

#### CONCLUDING REMARKS

This brief review has primarily surveyed analytical approaches to fatigue. To recognize that this review is not in depth, one has only to note that over 125 papers on various aspects of the subject were presented at a recent conference. However, much of the nature of the current approach, if not all of the specific detail has been indicated. In recent years, improvements in analysis rather than improvements in materials resistance to fatigue have been obtained. While it is to be hoped that material improvements will be forthcoming, thus far, this goal has been a difficult one to attain. Perhaps more modest goals such as reduction in scatter, in notch sensitivity, and in environmental sensitivity may be more realizable and useful than improvements in average properties. At any rate, a wider dissemination of information already available about fatigue design would constitute a positive step toward the elimination of fatigue failures.

### **ACKNOWLEDGMENT**

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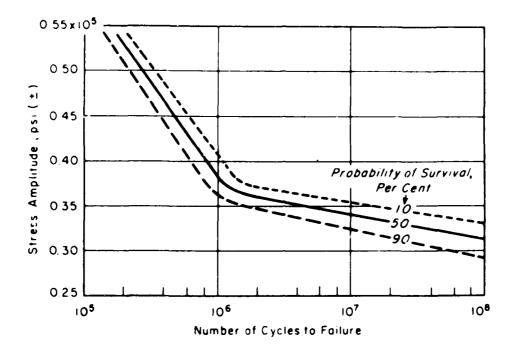


Fig. 1. Stress-amplitude - fatigue life - probability (SNP) plot for an aluminum alloy(6).

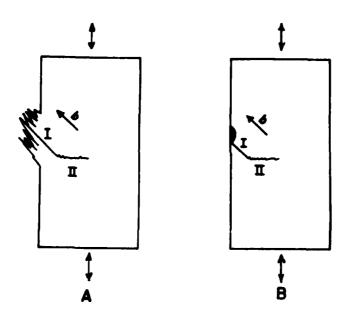


Fig. 2. Fatigue crack initiation sites in (A) a slip band, and (B) at an inclusion. The STAGE I of crack growth is along primary slip planes; the STAGE II of crack growth is perpendicular to the principal tensile stress axis.

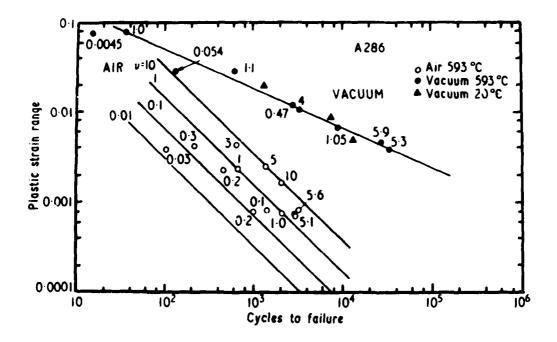


Fig. 3. Plastic strain range vs. fatigue life for A286 in air and vacuum at 593°C. Numbers adjacent to test points indicate frequency in cycle/min. After Coffin (7).

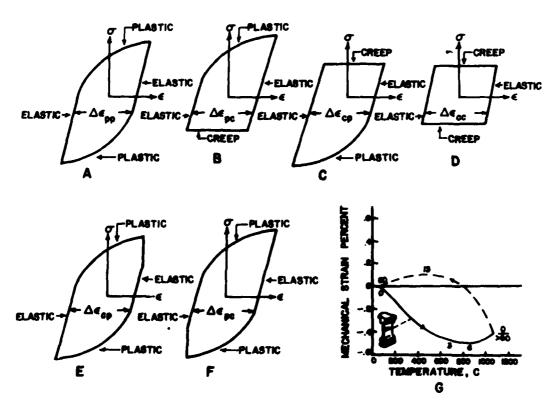


Fig. 4. A-F indicate a variety of hysteresis loop shapes which can be developed at elevated temperatures as a result of plastic deformation, creep and stress relaxation. G indicates a thermal fatigue cycle in which both temperature and strain vary.

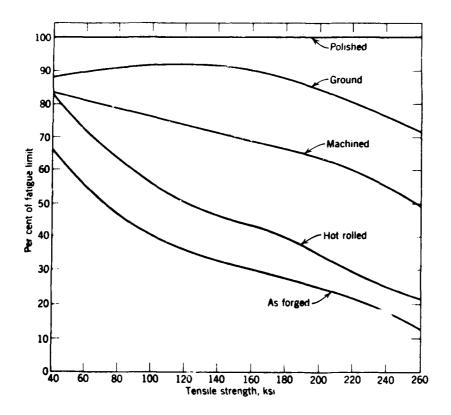


Fig. 5. Effect of surface on fatigue as a function of tensile strength of steel (10).

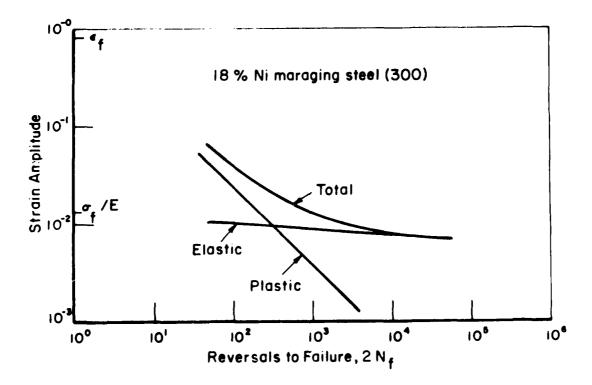


Fig. 6. Low cycle fatigue of a maraging steel as a function of elastic, plastic, and total strain amplitudes (12).

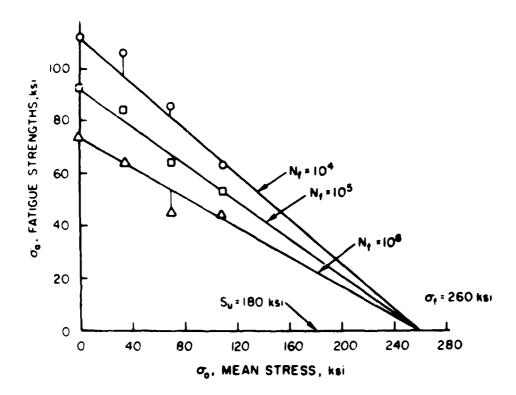


Fig. 7. A Goodman diagram for a steel of 180 ksi tensile strength  $^{(13)}$ .

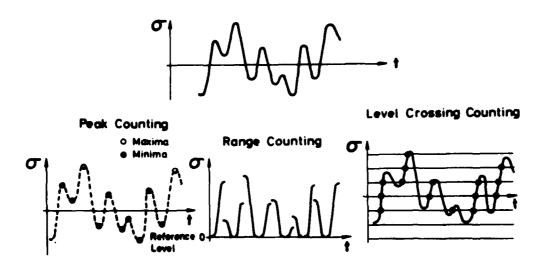


Fig. 8. Example of three basic types of counting methods in the analysis of variable amplitude loading (14).

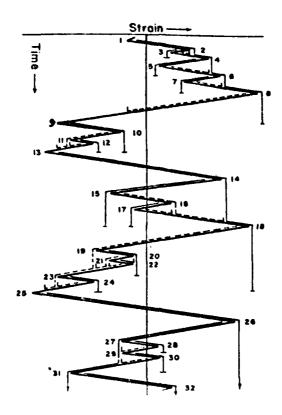


Fig. 9. Example of range pair counting method. (15) Solid and dashed ranges are paired to form cycle of corresponding mean strain.

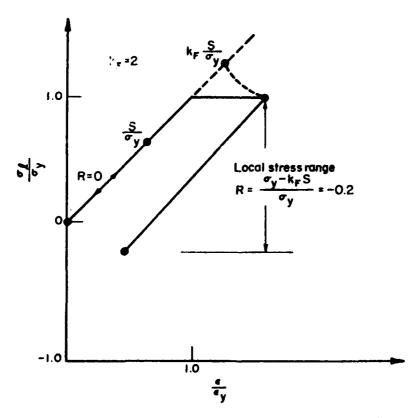


Fig. 10. Stress range at root of notch as influenced by localized yielding. Stresses and strains normalized with respect to yield stresses and strains, respectively. Applied R ratio equal to zero; fatigue notch factor, Kp=2.

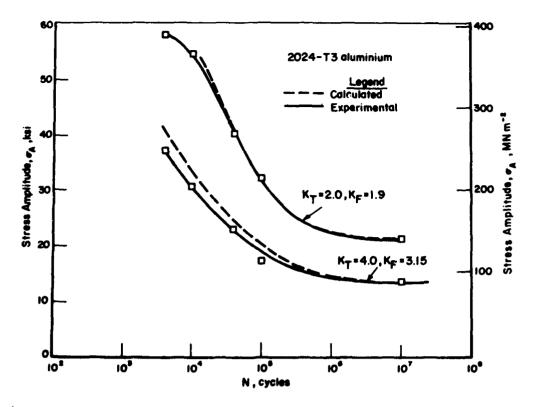


Fig. 11. Comparison between experimental and predicted fatigue lives for two stress raisers in an aluminum alloy.

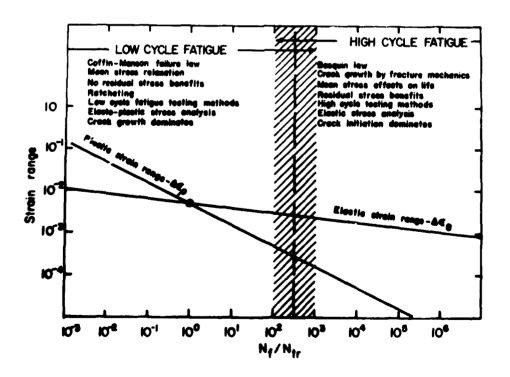


Fig. 12. Characteristics of low and high cycle fatigue ranges as a function of the transition life. After Ooffin(20).

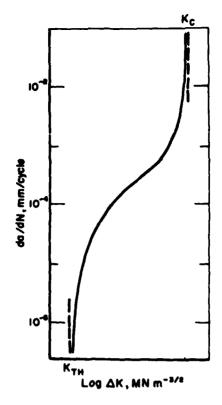


Fig. 13. Schematic dependence of the rate of crack growth,  $\frac{da}{dN}$  , on the stress intensity factor K.

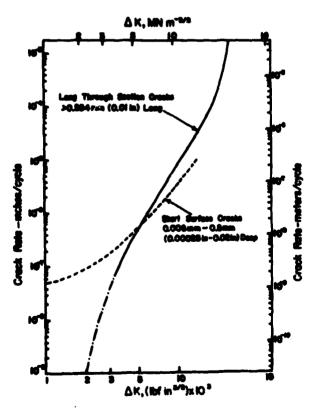


Fig. 14. Comparison of crack growth rate as a function of K for short surface cracks and through cracks. After Pearson (22).

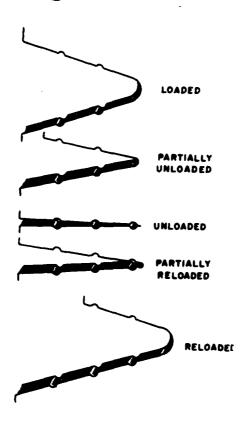


Fig. 15. Schematic of the fatigue crack growth process during a loading cycle in the absence of static modes of separation (23).

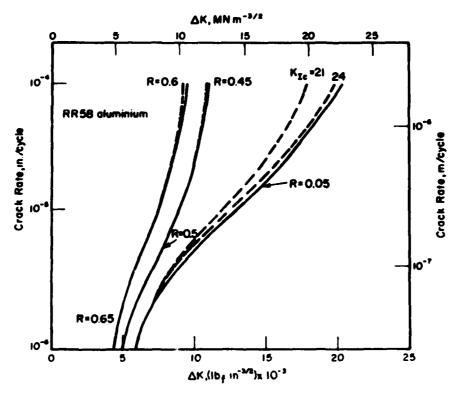


Fig. 16. Dependence of the rate of crack growth on mean stress and the range of the stress intensity factor for a low toughness aluminum alloy. Experimental results after Pearson (25) shown in solid lines; Eq. 14, dashed lines.

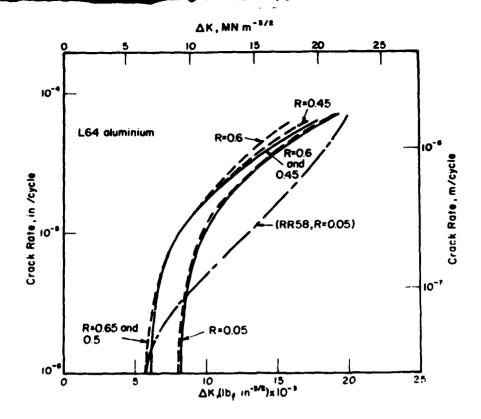


Fig. 17. Dependence of the rate of crack growth on mean stress and the range of the stress intensity factor for a high toughness aluminum alloy. Experimental results after Pearson (25) shown in solid lines; Eq. 14, dashed lines.

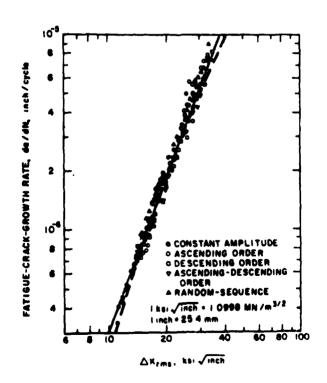


Fig. 18. Dependence of the rate of crack growth for variable amplitude loading on  $\Delta K_{rms}$ . Eq. 15 represented by the solid line; Eq. 17 represented by the dashed line. Data after Barsom (24).

### INSTITUTE OF MATERIALS SCIENCE

The Institute of Materials Science was established at The University of Connecticut in 1966 in order to promote the various fields of materials science. To this end, the State of Connecticut appropriated \$5,000,000 to set up new laboratory facilities, including approximately \$2,150,000 for scientific equipment. In addition, an annual budget of several hundred thousand dollars is provided by the State Legislature to support faculty and graduate student salaries, supplies and commodities, and supporting facilities such as various shops, technicians, secretaries, etc.

IMS fosters interdisciplinary graduate programs on the Storrs campus and at present is supporting five such programs in Alloy Physics, Biomaterials, Crystal Science, Metallurgy, and Polymer Science. These programs are directed toward training graduate students while advancing the frontiers of our knowledge in technically important areas.